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Bond Energy of Graph

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Abstract— In this paper, we introduce the bond matrix and bond spectrum of a simple connected graph G. Then we establish the bond energy of G .Then we analyze the bond energy $EM_{ED}(G)$ of a simple connected graph and establish upper and lower bounds. In this paper we analyze bond energy for some standard family of graphs which include complete graphs, family of regular graphs, cycle graphs star graphs and some special graphs. Then we present the bounds of bond energy of a graph.

Index Terms— Bond matrix, Bond spectrum of a graph, Bond energy of a graph, Bounds for bound energy.

1 Introduction

In this paper, let $G=(V,E)$ be a simple graph i.e. G contains no loops, multiple and directed edges.Conventionally we denote the number of vertices by $n=|V|$ and number of edges by $m=|E|$ of a connected graph G. For any vertex v_i in G, suppose $\delta(v_i)$ and $\Delta(v_i)$ is the minimum and maximum degree of a graph G. In this article, a graph is said to be a $k-$ regular if degree of each vertex v in G, that is $d(v)=k$. For simplicity, let K_n, C_n and $K_{r,r}$ be the complete, cyclic and bipertite graphs. For nomenclature of different terminology we refer to readers to see books [1, 2].

The idea of a graph is introduced by I. Gutman [3]. For any graph $G=(V,E)$, where $|V|=n$ is vertices and $|E|=m$ is edges of G. Let $A(G)$ be the adjacency matrix $A(G)=[a_{ij}]$ of G such that

$$a_{ij}=\begin{cases}1 & \text{if } v_i \text{ and } v_j \text{ are adjacent}\\ 0 & \text{otherwise}\end{cases}$$

where v_i and v_j are ith and jth vertices of G. Let $\lambda_1,\lambda_2,...,\lambda_n$ be the eigenvalues of adjacency matrix $A(G)$. Furthermore assume that these eigenvalues are non-increasing order and distinct for some $l\le n$ such that $\lambda_l\le\lambda_{l-1}\le\cdots\le\lambda_2\le\lambda_1$. Let $m_1,m_2,...,m_l$ be the multiplicity of $\lambda_1,\lambda_2,\cdots,\lambda_l$ respectively. Then the spectrum of graph G is defined as:

$$Spec(G)=\begin{pmatrix}\lambda_1 & \lambda_2 & \cdots & \lambda_l\\ m_l & m_2 & \cdots & m_l\end{pmatrix}$$

Since the $A(G)$ is real symmetric matrix with zero trace, so the eigenvalues of G are reals and sum of all eigenvalues of G zero.

For a given graph G with n vertices the energy of a graph is defined, denoted $E(G)$, by

$$E(G)=\sum_{i=1}^{n}|\lambda_i|.$$

As we know that the energy of a graph is independent of isolated vertices implying, in particularly $m\ge n/2$. The idea of graph energy originate from in chemistry, for some numerical quantities like heat of formation of a hydrogen, are related to total $\pi-$ electron energy which construct by energy of an appropriate molecular graph as in [4].

To introduce the bond energy, we define some basic parameters such as walk and trail. A walk is a sequence of alternating vertices and edges such as $v_1,m_1,v_2,m_2,\ldots,m_k,v_k$ for each edge $m_1,m_2,...,m_k$. The length of this walk is k. A trail is a walk with no repeated edges. This means that we can repeat a vertex to get different number of trails.

2 Bond Matrix and Bond Energy

Consider a simple connected graph G of order n. Let $V=\{v_1,v_2,\ldots,v_n\}$ be the vertex set and $E=\{m_1,m_2,\ldots,m_n\}$ be the edge set in graph G.The maximum number of edge-disjoint trails from a vertex v_i to v_j in graph $G=(V,E)$ is denoted by Ω_{ij}. Since each such trail must contain an edge from every v_i-v_j edge separating set. Note that in such edge separating set, there is no even a single edge is repeated. The bond matrix $M_{ED}(G)=[M_{ij}]$ of a graph G is defined as:

$$M_{ij}=\begin{cases}\Omega_{ij} & \text{if there exists trail between } v_i \text{ and } v_j\\ 0 & otherwise\end{cases}$$

where Ω_{ij} is the maximum number of edge-disjoint trails from v_i to v_j. Bond matrix provides the importance of the connectivity of a vertex to the other vertex. In particularly, the entries in bond matrix show the strength of two vertices. It plays an important role in chemistry that is how much two atoms or molecules are bounds. Similarly we can extract this idea into real life problems like a person is attached to how many others

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people by some refrence people. The characteristics polynomial of bond matrix $M_{ED}(G)$ is defined by

$$P(G,\lambda)=\det(\lambda I - M_{ED}(G)),$$

where I is the identity matrix with dimension $(n\times n)$. Since the bond matrix $M_{ED}(G)$ is real and symmetric with zero trace, so its eigenvalue are real and the sum of its eigenvalues is equal to zero. Assume that the eigenvalues are in non-increasing order $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_n$. Thus the spectrum of graph is

$$M_{ED}Spec(G)=\begin{pmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_n \\ m_1 & m_2 & \cdots & m_n \end{pmatrix}$$

The bound energy of a graph is the sum of absolute eigenvalues of bond matrix $M_{ED}(G)$ of a graph G .Then we can define bond energy as follows:

$$EM_{ED}(G)=\sum_{i=1}^{n}|\lambda_i|.$$

Now we analyze bond energy by some examples given below.

2.1 A motivation example

Consider a graph G with vertices a,b,c,d,e,f as shown in Figure.1.

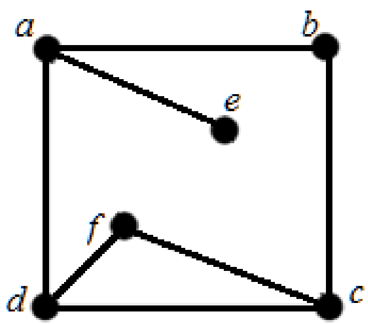

Figure.1: G_1

The bound matrix of G_1 is given as:

$$M_{ED}(G_1)=\begin{bmatrix} 1 & 2 & 2 & 2 & 1 & 2 \\ 2 & 1 & 2 & 2 & 1 & 2 \\ 2 & 2 & 1 & 3 & 1 & 2 \\ 2 & 2 & 3 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 2 & 2 & 2 & 2 & 1 & 1 \end{bmatrix}$$

The characteristics polynomial of $M_{ED}(G_1)$ is

$$P(G_1,\lambda)=\det(\lambda I - M_{ED}(G_1))$$

$$=\begin{vmatrix} \lambda-1 & -2 & -2 & -2 & -1 & -2 \\ -2 & \lambda-1 & -2 & -2 & -1 & -2 \\ -2 & -2 & \lambda-1 & -3 & -1 & -2 \\ -2 & -2 & -3 & \lambda-1 & -1 & -2 \\ -1 & -1 & -1 & -1 & \lambda & -1 \\ -2 & -2 & -2 & -2 & -1 & \lambda-1 \end{vmatrix}$$

$$=\lambda^6-5\lambda^5-40\lambda^4-81\lambda^3-71\lambda^2-28\lambda-4$$

$$=(\lambda+1)^2(\lambda+2)(\lambda^3-9\lambda^2-9\lambda+2)$$

Then the bond spectrum of G_1 is

$$M_{ED}Spec(G_1)=\begin{pmatrix} 9.9269 & -0.3480 & -0.5788 & -1 & -2 \\ 1 & 1 & 1 & 2 & 1 \end{pmatrix}$$

Therefore the bond energy of G_1 is

$$EM_{ED}(G_1)=14.85.$$

2.2 A motivation example

Consider a graph G_2 as shown in Figure.2

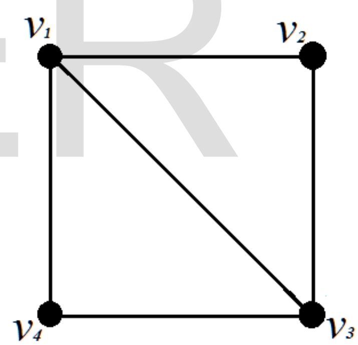

Figure.2: G_2

The bound matrix of G_2 is given as:

$$M_{ED}(G_2)=\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 1 & 2 & 2 \\ 3 & 2 & 1 & 2 \\ 2 & 2 & 2 & 1 \end{bmatrix}$$

The characteristics polynomial of $M_{ED}(G_2)$ is

$$P(G_1,\lambda)=\det(\lambda I - M_{ED}(G_2))$$

$$=\begin{vmatrix} \lambda-1 & -2 & -3 & -2 \\ -2 & \lambda-1 & -2 & -2 \\ -3 & -2 & \lambda-1 & -2 \\ -2 & -2 & -2 & \lambda-1 \end{vmatrix}$$

$$=\lambda^4-4\lambda^3-23\lambda^2-26\lambda-8$$

$$= (\lambda+2)(\lambda+1)(\lambda^2-5\lambda-10)$$

Then the bond spectrum of G_2 is

$$M_{ED}Spec(G_1) = \begin{pmatrix} \frac{7+\sqrt{65}}{2} & \frac{7-\sqrt{65}}{2} & -1 & -2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Therefore the bond energy of G_2 is

$$EM_{ED}(G_2) = 11.06\,.$$

3 Bond Energy of Some Standard Graphs

In this section we illustrate some well-known families of graphs and find their exact bond matrices and bond energies.

Theorem 3.1 *Let $K_{1,n-1}$ be a star graph of order $n \geq 3$ then the bond energy of star graph is*

$$EM_{ED}(K_{1,n-1}) = n\,.$$

Proof. Suppose that $K_{1,n}$ be a star graph and let $M_{ED}(K_{1,n})$ be the bond matrix of $K_{1,n}$.

$$M_{ED}(K_{1,n}) = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & 1 & 1 & 1 & \cdots & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & \cdots & 1 & 0 \end{pmatrix}_{n\times n}$$

The characteristic polynomial of $M_{ED}(K_{1,n})$ is

$$P(K_{1,n},\lambda) = \begin{vmatrix} \lambda & -1 & -1 & -1 & -1 & \cdots & -1 & -1 \\ -1 & \lambda & -1 & -1 & -1 & \cdots & -1 & -1 \\ -1 & -1 & \lambda & -1 & -1 & \cdots & -1 & -1 \\ -1 & -1 & -1 & \lambda & -1 & \cdots & -1 & -1 \\ -1 & -1 & -1 & -1 & \lambda & \cdots & -1 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & -1 & -1 & -1 & -1 & \cdots & \lambda & -1 \\ -1 & -1 & -1 & -1 & -1 & \cdots & -1 & \lambda \end{vmatrix}$$

$$= (\lambda - 1(n-1))(\lambda+1)^{n-1}$$

Hence, the bond spectrum of $K_{1,n}$ is

$$M_{ED}Spec(K_{1,n}) = \begin{pmatrix} n-1 & -1 \\ 1 & n-1 \end{pmatrix}$$

Therefore the bond energy of star $K_{1,n}$ is

$$EM_{ED}(K_{1,n}) = n\,.$$

Remark: The bond energy of a star is the order of size of star graph.

Theorem 3.2 *Let C_n be a cycle graph of order n for $n \geq 3$. Then the bond energy of cycle graph C_n is*

$$EM_{ED}(C_n) = n\,.$$

Proof. Consider a cycle graph C_n with vertex set $v_1, v_2, \ldots, v_n$, for $n \geq 3$. Then the bond matrix is defined as:

$$M_{EI}(C_n) = \begin{pmatrix} 1 & 2 & 2 & 2 & 2 & \cdots & 2 & 2 \\ 2 & 1 & 2 & 2 & 2 & \cdots & 2 & 2 \\ 2 & 2 & 1 & 2 & 2 & \cdots & 2 & 2 \\ 2 & 2 & 2 & 1 & 2 & \cdots & 2 & 2 \\ 2 & 2 & 2 & 2 & 1 & \cdots & 2 & 2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & 2 & 2 & 2 & 2 & \cdots & 1 & 2 \\ 2 & 2 & 2 & 2 & 2 & \cdots & 2 & 1 \end{pmatrix}_{n\times n}$$

The characteristic polynomial of cycle graph is

$$P(C_n,\lambda) = \begin{vmatrix} \lambda-1 & -2 & -2 & -2 & -2 & \cdots & -2 & -2 \\ -2 & \lambda-1 & -2 & -2 & -2 & \cdots & -2 & -2 \\ -2 & -2 & \lambda & -2 & -2 & \cdots & -2 & -2 \\ -2 & -2 & -2 & \lambda-1 & -2 & \cdots & -2 & -2 \\ -2 & -2 & -2 & -2 & \lambda-1 & \cdots & -2 & -2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -2 & -2 & -2 & -2 & -2 & \cdots & \lambda-1 & -2 \\ -2 & -2 & -2 & -2 & -2 & \cdots & -2 & \lambda-1 \end{vmatrix}$$

$$= (\lambda - 2(n-1))(\lambda+2)^{2(n-1)}$$

Hence, the bond spectrum is

$$M_{ED}Spec(C_n) = \begin{pmatrix} 2(n-1) & -2 \\ 1 & n-1 \end{pmatrix}$$

Therefore, the bond energy of cycle graph is

$$EM_{ED}(C_n) = 2n \qquad \square$$

Remark: The bond energy of cycle graph is order of cycle.

Theorem 3.3 *Let A_r be an antiprism graphs for $n = 2r$ and $r \geq 3$. Then the bond energy of antiprism graph*

$$EM_{ED}(A_r) = 4n.$$

Proof. Let A_r be a antiprism graph of order n such that $n = 2r$ for $r \geq 3$. Then the bond matrix is

$$M_{ED}(A_r) = \begin{pmatrix} 2 & 4 & 4 & 4 & 4 & \cdots & 4 & 4 \\ 4 & 2 & 4 & 4 & 4 & \cdots & 4 & 4 \\ 4 & 4 & 2 & 4 & 4 & \cdots & 4 & 4 \\ 4 & 4 & 4 & 2 & 4 & \cdots & 4 & 4 \\ 4 & 4 & 4 & 4 & 2 & \cdots & 4 & 4 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 4 & 4 & 4 & 4 & 4 & \cdots & 2 & 4 \\ 4 & 4 & 4 & 4 & 4 & \cdots & 4 & 2 \end{pmatrix}_{n \times n}$$

The characteristic polynomial is

$$P(A_r, \lambda) = \begin{vmatrix} \lambda-2 & 4 & 4 & 4 & 4 & \cdots & 4 & 4 \\ 4 & \lambda-2 & 4 & 4 & 4 & \cdots & 4 & 4 \\ 4 & 4 & \lambda-2 & 4 & 4 & \cdots & 4 & 4 \\ 4 & 4 & 4 & \lambda-2 & 4 & \cdots & 4 & 4 \\ 4 & 4 & 4 & 4 & \lambda-2 & \cdots & 4 & 4 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 4 & 4 & 4 & 4 & 4 & \cdots & \lambda-2 & 4 \\ 4 & 4 & 4 & 4 & 4 & \cdots & 4 & \lambda-2 \end{vmatrix}$$

$$= (\lambda - 2(4r-1))(\lambda+2)^{2r-1}$$

The bond spectrum of antiprism graph is

$$M_{ED}Spec(A_r) = \begin{pmatrix} 2(4r-1) & -2 \\ 1 & 2r-1 \end{pmatrix}$$

Since $n = 2r$ and therefore the bond energy of antiprism graph A_r is

$$EM_{ED}(A_r) = 8r = 4n. \qquad \square$$

Theorem 3.4 *Suppose K_n be a complete graph with $n \geq 2$. The bond energy of complte graph is*

$$EM_{ED}(K_n) = n^2 - n.$$

Proof. Let K_n be a complete graph of oder n such that $n \geq 2$. Then the bond matrix is given as:

$$M_{ED}(K_n) = \begin{pmatrix} f & n-1 & n-1 & n-1 & n-1 & \cdots & n-1 & n-1 \\ n-1 & f & n-1 & n-1 & n-1 & \cdots & n-1 & n-1 \\ n-1 & n-1 & f & n-1 & n-1 & \cdots & n-1 & n-1 \\ n-1 & n-1 & n-1 & f & n-1 & \cdots & n-1 & n-1 \\ n-1 & n-1 & n-1 & n-1 & f & \cdots & n-1 & n-1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ n-1 & n-1 & n-1 & n-1 & n-1 & \cdots & f & n-1 \\ n-1 & n-1 & n-1 & n-1 & n-1 & \cdots & n-1 & f \end{pmatrix}_{n \times n}$$

The characteristic polynomial of bond matrix $M_{ED}(K_n)$ is

$$P(K_n, \lambda) = \begin{vmatrix} \lambda-f & -(n-1) & -(n-1) & -(n-1) & -(n-1) & \cdots & -(n-1) & -(n-1) \\ -(n-1) & \lambda-f & -(n-1) & -(n-1) & -(n-1) & \cdots & -(n-1) & -(n-1) \\ -(n-1) & -(n-1) & \lambda-f & -(n-1) & -(n-1) & \cdots & -(n-1) & -(n-1) \\ -(n-1) & -(n-1) & -(n-1) & \lambda-f & -(n-1) & \cdots & -(n-1) & -(n-1) \\ -(n-1) & -(n-1) & -(n-1) & -(n-1) & \lambda-f & \cdots & -(n-1) & -(n-1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -(n-1) & -(n-1) & -(n-1) & -(n-1) & -(n-1) & \cdots & \lambda-f & -(n-1) \\ -(n-1) & -(n-1) & -(n-1) & -(n-1) & -(n-1) & \cdots & -(n-1) & \lambda-f \end{vmatrix}$$

$$= (\lambda - (n(n-1) - d))(\lambda - d)^{n-1}$$

where the f and d is defined as:

$$f = \begin{cases} \dfrac{n-1}{2} & \text{if } n \text{ is odd} \\ \dfrac{n-2}{2} & \text{if } n \text{ is even} \end{cases}$$

$$d = \begin{cases} \dfrac{n}{2} & \text{if } n \text{ is even} \\ \dfrac{n-1}{2} & \text{if } n \text{ is odd} \end{cases}$$

The bond spectrum of complete graph K_n is

$$M_{ED}Spec(K_n) = \begin{pmatrix} n^2 - n - d & -d \\ 1 & n-1 \end{pmatrix}$$

Therefore, the bond energy of complete graph is

$$EM_{ED}(K_n) = n^2 - n. \qquad \square$$

4 Bond Energy for Some Special Graphs

4.1 Wagner graph [5]

Consider a Wagner graph M_8 with vertex set $v_1, v_2, \ldots, v_8$. The bond matrix is defined as

$$M_{ED}(M_8)=\begin{pmatrix}1&3&3&3&3&3&3&3\\3&1&3&3&3&3&3&3\\3&3&1&3&3&3&3&3\\3&3&3&1&3&3&3&3\\3&3&3&3&1&3&3&3\\3&3&3&3&3&1&3&3\\3&3&3&3&3&3&1&3\\3&3&3&3&3&3&3&1\end{pmatrix}_{8\times 8}$$

The characteristic polynomial of bond matrix $M_{ED}(M_8)$ is

$$P(M_8,\lambda)=\begin{vmatrix}\lambda-1&-3&-3&-3&-3&-3&-3&-3\\-3&\lambda-1&-3&-3&-3&-3&-3&-3\\-3&-3&\lambda-1&-3&-3&-3&-3&-3\\-3&-3&-3&\lambda-1&-3&-3&-3&-3\\-3&-3&-3&-3&\lambda-1&-3&-3&-3\\-3&-3&-3&-3&-3&\lambda-1&-3&-3\\-3&-3&-3&-3&-3&-3&\lambda-1&-3\\-3&-3&-3&-3&-3&-3&-3&\lambda-1\end{vmatrix}$$
$$=(\lambda-22)(\lambda+2)^7$$

The bond spectrum of Wagner graph is

$$M_{ED}Spec(M_8)=\begin{pmatrix}22&-2\\1&7\end{pmatrix}$$

The bond energy of Wagner graph is

$$EM_{ED}(M_8)=24\,.$$

4.2 Chvátal graph

Let B_{12} be a Chvátal graph with vertices $v_1,v_2,\ldots,v_{12}$. The bond matrix is

$$M_{ED}(B_{12})=\begin{pmatrix}2&4&4&4&4&\cdots&4&4\\4&2&4&4&4&\cdots&4&4\\4&4&2&4&4&\cdots&4&4\\4&4&4&2&4&\cdots&4&4\\4&4&4&4&2&\cdots&4&4\\\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\4&4&4&4&4&\cdots&2&4\\4&4&4&4&4&\cdots&4&2\end{pmatrix}_{12\times 12}$$

The characteristic polynomial of bond matrix $M_{ED}(B_{12})$ is

$$P(B_{12},\lambda)=\begin{vmatrix}\lambda-2&-4&-4&-4&-4&\cdots&-4&-4\\-4&\lambda-2&-4&-4&-4&\cdots&-4&-4\\-4&-4&\lambda-2&-4&-4&\cdots&-4&-4\\-4&-4&-4&\lambda-2&-4&\cdots&-4&-4\\-4&-4&-4&-4&\lambda-2&\cdots&-4&-4\\\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\-4&-4&-4&-4&-4&\cdots&\lambda-2&-4\\-4&-4&-4&-4&-4&\cdots&-4&\lambda-2\end{vmatrix}$$
$$=(\lambda-46)(\lambda+2)^{11}$$

The bond spectrum of Wagner graph is

$$M_{ED}Spec(B_{12})=\begin{pmatrix}46&-2\\1&11\end{pmatrix}$$

The bond energy of Wagner graph is

$$EM_{ED}(B_{12})=48\,.$$

4.3 Petersen graph

Consider a petersen graph P_{10} with vertex set $v_1,v_2,\ldots,v_{10}$. The Petersen graph is a regular in which the degree of every vertex is three. The bond matrix is defined as

$$M_{ED}(P_{10})=\begin{pmatrix}1&3&3&3&3&\cdots&3&3\\3&1&3&3&3&\cdots&3&3\\3&3&1&3&3&\cdots&3&3\\3&3&3&1&3&\cdots&3&3\\3&3&3&3&1&v&3&3\\\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\3&3&3&3&3&\cdots&1&3\\3&3&3&3&3&\cdots&3&1\end{pmatrix}_{10\times 10}$$

The characteristic polynomial of bond matrix $M_{ED}(P_{10})$ is

$$P(P_{10},\lambda)=\begin{vmatrix}\lambda-1&-3&-3&-3&-3&-3&-3&-3\\-3&\lambda-1&-3&-3&-3&-3&-3&-3\\-3&-3&\lambda-1&-3&-3&-3&-3&-3\\-3&-3&-3&\lambda-1&-3&-3&-3&-3\\-3&-3&-3&-3&\lambda-1&-3&-3&-3\\-3&-3&-3&-3&-3&\lambda-1&-3&-3\\-3&-3&-3&-3&-3&-3&\lambda-1&-3\\-3&-3&-3&-3&-3&-3&-3&\lambda-1\end{vmatrix}$$
$$=(\lambda-22)(\lambda+2)^7$$

The bond spectrum of petersen graph P_{10} is

$$M_{ED}Spec(P_{10})=\begin{pmatrix}22&-2\\1&7\end{pmatrix}$$

The bond energy of petersen graph P_{10} is

$$EM_{ED}(P_{10}) = 24 .$$

5 Bounds for Bond Energy

In this section we demonstrate the bounds for bond energy of a graph G . The bounds splits into two form an upper bound and lower bound which is established in following theorems.

Theorem 5.1 *Let G be a simple $r-$regular graph of order n . Then*

$$0 \le EM_{ED}(G) \le nr$$

Proof. Let G be a $r-$regular graph. If $r = 0$ then the graph G contains no edge, so there is no trail to move on. Then the bond matrix becomes actually a null matrix **0**. Then the eigenvalues of such null matrix are all equal to zero. Therefore, $EM_{ED}(G) = 0$. Since the bond energy is the sum of absolute of eigenvalues of bond matrix. Thus bond energy is non-negative. Hence,

$$EM_{ED}(G) \ge 0 .$$

Assume that we have a $r-$regular graph G of degree r . Let n be the order of graph with vertices $v_1, v_2, \ldots, v_n$ .Then the degree of all vertices are r i.e. $d_1 = d_2 = \cdots = d_n = r$, where d_i is the degree of vetex v_i . Then the all non-zeros of bond matrix are r with multiplicity n . Therefore, the bond energy of r-regular graph is, $EM_{ED}(G) < nr$. The $EM_{ED}(G) = nr$, equality holds if we add all missing edges in regular graph until the there is no capicty to add more edges in graph G . Therefore,

$$0 \le EM_{ED}(G) \le nr . \qquad \square$$

6 Conclusion

In this article, we established a new matrix of a simple connected graph G known as bond matrix $M_{ED}(G)$ and calculate its energy named as bound energy of a graph G . The bond matrix depends on the connectitvity of a graph. To motivate the concept we give its some numerical example. Some of the properties like bond matrix and energy of complete graph, bipartite graph, star graph and cycle graph are discussed. The upper and lower bounds of bond matrix are obtained. This is true, if way say there are many applications of bonds matrix in graph theory, electrical engineering mathematical algebra as well as in other areas.

Open Problems

1- Find bond matrix and bond energy of some other family of graphs.
2- Find other bounds for the bond energy of a connected graph.

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